A GRAPH THEORETIC APPROACH TO MULTI-ROBOT FORMATION CONTROL

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Oscar Ruiz

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TITLE: A Graph Theoretic Approach to Multi-Robot Formation Control

AUTHOR: Oscar Ruiz

DATE SUBMITTED: June 2017

COMMITTEE CHAIR: HelenYu, Ph.D.

Professor of Electrical Engineering

COMMITTEE MEMBER: Foaad Khosmood, Ph.D.

Professor of Computer Engineering

COMMITTEE MEMBER: Hisham Assal, Ph.D.

Lecturer of Computer Engineering

**ABSTRACT**

There are a variety of methods proposed for establishing a distributed controller for a multi-robot system. This thesis will focus on a graph theoretic approach to multi-robot formation control. We will compare an absolute position based algorithm previously developed by others, and an extension to the algorithm which removes dependence on an absolute reference frame. These robots will have access to distance information between robots as well as the direction of the neighboring robots relative to a given robot’s heading. The system will be represented as Graph and it will be shown how it can be described by the Graph Laplacian Matrix. The edges of the graph will be assigned a potential energy function and the total energy of the system will be reduced by formulating the system equation in the form of a gradient descent. We will also use the graph structure to specify robot formation and demonstrate the relationship between the Graph Laplacian Matrix and robot formations.

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**Chapter 1: Introduction**

Swarm robotics is a method of coordination for multi robot systems, which are composed of large numbers of mostly simple robots [5]. The desired outcome is a collective behavior emerging from the interaction between the robots and their environment [12]. The robots are also autonomous, not controlled centrally and capable of local communication [13]. Swarm robotics has become a major research area since the 1980’s and new solution approaches are being developed and validated [13].  This thesis will focus on inter-robot motion coordination and some of the algorithms used for this task.

There are several active areas of research in the field of swarm robotics [5]. Swarm robotics and swarm intelligence is inspired by the decentralized organizational patterns found in the study of biology [10]. There are current attempts to adapt these natural processes to robot control and algorithm design. There is also on-going research in robotic swarm mobility, environment manipulation and reconfigurable robotics [10]. There are several key advantages to implementing a multi robot swarm as opposed to a single robotic platform. For situations where tasks can be decomposed into smaller subtasks, swarms can accomplish a given task more rapidly than a single robot by dividing the work into smaller parts and carrying out the subtasks in parallel [14]. Also, a robotic swarm can be designed in such a way that no single point of failure exists within the system. In other words, if an individual within the swarm becomes inoperable, the swarm itself should continue carrying out the tasks without interference. Related to this is the idea of scalability and locality. If more robots are added to the swarm, the individual’s already participating in the swarm should not be affected by the addition of new members. Also, individual robots within the swarm should be in communication with robots in their local vicinity and not with individuals that are far away and out of sight.

This is an important problem solve because there are situations where it is advantageous to have multiple agents collaborating to accomplish a task as opposed to a single system. For example, the exploration of Mars could be done more rapidly with many smaller, simpler and cheaper robots as opposed one large expensive one. The multi-robot system could explore a larger area and be less vulnerable mission failure if one of the members of the system become inoperable. Another valuable application of swarm intelligence in robotics is in search and rescue. Small robots, with swarming capabilities can search destroyed building for survivors by being able to reach places that are unreachable by rescuers. There are also applications in formation control for satellite clusters. The Terrestrial Planet Finder is a proposed deep space interferometer composed of multiple telescopes which stay in formation. Communication between the platforms is essential and the control of the distances between the telescope is important for correction operation of the telescope.

The project will study robotic swarm motion coordination and how robots within a swarm calculate and readjust their positions with respect to each other using a Particle Swarm Optimization(PSO) algorithm. The rendezvous problem will be modeled and solved using PSO and modifications will be introduced to allow for greater control over a particle swarm. The goal of this project is to develop a particle swarm optimization algorithm, which allows for semi-autonomous control of a multi-agent system. This is needed in the control of robotic swarms where we have a distributed network of simple robots, which have limited communication capabilities. We can start by determining the distances between all the individuals and update their positions iteratively by subtracting from their current positions. The end effect would be the convergence of the swarm on one point.  However, this is unrealistic for a physical robotic swarm because if would cause a collision at the convergence point.  We can introduce weights to the algorithm to slow down and control the rate of convergence.  We will describe this procedure more thoroughly in the Background section.

**Chapter 2: Literature Review**

We found several Graph Theoretic Methods for establishing multi-agent control systems as well state switching algorithms such as [4] as well as swarming method based on real time learning using an embedded neural network [5]. Since our method is based on Graph Theory we reference those paper heavily while also comparing them to alternative methods.

The model described in this thesis is based on the decentralized control strategy from [1]. The paper describes the graph based structure of a multi-agent system in terms of edges and vertices as ours does. Our method diverges from the one found in [1] in the definition of the Graph Laplacian Matrix. They define their L matrix as the Gramian Matrix of the Incidence Matrix while we describe our L matrix as the differences between the Degree and Adjacency matrix. Both definitions are equivalent. Our definition of the L matrix has the advantage of giving as access to the Adjacency and Degree values directly which become useful for building prescribed formations in section 3.4.

Our definition for the Graph Laplacian is found in [2]. The paper also provides a weighted version of the graph Laplacian as described in this thesis and introduces a theoretical framework for controlling graph connectivity in mobile robot networks. [2] uses a finally trigonometric function as their energy weights instead of our parabolic function. We chose a parabolic function because of its ease of implementation and to avoid having to fine tune. They derived their weights functions in the exact same as we did however. [2] also introduces an obstacle avoidance augmentation of their algorithm which is useful for guiding the formation around and through a space filled external objects. I have done some exploration into their method have not tested fully as of (March 10th 2017).

This paper [3] considers a group of wheeled robots with non-holonomic constraints and a method for rendezvous at a common specified point with prescribed orientation while also maintaining network connectivity and ensuring collision avoidance within robots. This attempt is different from the attempt in this thesis because they do not invoke Graph Theory and instead derive their algorithm from dynamics. This will be used as an example for comparing our method to an alternative approach.

This paper [4] focuses on foraging as a multi-robot task and present two distributed foraging algorithm each of which performs best for different food distributions and locations. A third algorithm with the ability switch between the two previous algorithms is also presented. The method is based on state switching of each individual robot based on the conditions around it. The first method is referred to as the Gradient Method where robots perform a random walk and switch to beacons at random moments in time. The beacons broadcast a number representing the gradient towards the nest. When the resource is found the bots know in which direction it is based on the gradient established. The second algorithm is based on a sweeping method where the bots form a line, hold in place based on a virtual force between them and circle around the nest to look for the resource. The virtual force described in this paper is similar to our energy function used in this thesis. The third algorithm switches between the two previous methods based on whether a given method is having success at finding the resource. We have already implemented algorithm similar to one described in [4], it is possible we might be able to merge the task of resource foraging using the formation control derived in this thesis.

In this paper [5] real time learning of multiple physical autonomous robots situated in a real dynamic environment is performed. Each robot had an onboard micro-controller where a simple artificial neural network ANN was embedded. The system was designed with consideration of the power and computational resource limitations of the robots. The robot ANN’s were started simultaneously and are allowed to run until the desired inter-robot distances are achieved. This method will be compared to our approach to compare convergence rate and complexity.

In this paper [6] the same problem of motion and network topology control in a group of mobile agents is explored but instead of assuming that the graph is connected, it is enforced through distributed topology control which decides on both direction and creation/deletion of links. So far (March 10th 2017) our algorithm assumes connectivity and has pre-supposed link deletions based on the formation we would like to achieve prior to starting the simulation. If we time allows it we will explore the method in [6] further and attempt to integrate it into our algorithm.

This paper [7] invokes low level trajectory controller based on dynamic feedback linearization. This approach was not used in this thesis but provides an example of what others have done for the problem of creating a high level multi agent algorithm and interfacing it with a low level controller for differential drive robots. This paper [8] is a Master’s thesis by a student from at Delft University of Technology. The paper discusses the control of robot swarms using Radio Signal Strength measurements on a group of six legged robots. The method used is not Graph Theoretic but is derived from the dynamics of the robots themselves. I am using [8] as a model for my own thesis and may not be included in the final literature review.

**Chapter 3: Theoretical Background**

***3.1 Graph Theory***

The following section presents a model of our multi-agent system in the context of Graph Theory and its mathematical structures based on [2] and [1]. We will be using the model described in the preliminary section of [1]. Graphs are composed of vertices connected by edges. Graphs can be directed or undirected. Undirected graphs do not make a distinction between two edges connected to the same two vertices while directed graphs do. We can represent our multi-agent system as a graph with the set of vertices as agents and the set of edges as the communication links between agents.



 is the Vertex set,

 is the Edge set.

For undirected graphs, the edge  connecting vertices is indistinguishable from edge . The graph representing our multi-robot system will be undirected because we do not care about the direction of communication between two robots. We are only concerned with the existence of connections between robots. We can define the Adjacency matrix of the graph as A. The Adjacency Matrix is a symmetrical matrix with diagonals equal to zero.







The Adjacency matrix defines which vertices are connected and which vertices are disconnected through their corresponding graph edges. The elementin A is equal to 1 when the edge exists and is equal to 0 when the connection edge does not exist. We can also define the Degree matrix of the graph. The Degree matrix is a diagonal matrix with the element  being the number of connections (the degree) to the vertex.



The diagonal elements of the degree matrix are also equal to the sum of the row of the corresponding Adjacency matrix. In other words:



Using the Degree Matrix and Adjacency Matrix we can define the Graph Laplacian as:



The Laplacian of the graph defines the uniqueness of the graph and can be used to define the state equation of our multi-robot system. We can write the state of the total system as:



In our system,is the position of an agent and n is the total number of agents. For simplicity, we will continue our analysis in one dimension but our derivation can be generalized to multiple dimensions. We can begin prescribing certain behaviors to our system do would allow us to more easily control their formation. We can begin with the elementary approach of driving two robots to single point.





The above equations of motion will cause the robots to drive towards each other until the distance between them is zero. If we extend this to  robots the equation of motion for a robot is:



The above equation assumes that the graph is fully connected. We can restrict the connections for each robot to a certain set which we will call its neighbors . A robot’s set of neighbors can be defined by proximity or through some other imposed means such as for the purposes of establishing a formation. When robot neighbors are defined, the equations of motion can be written as:



We can also write the consensus equation in terms of the of the number of connections to the given robot and the elements of the adjacency matrix that are non-zero.



The equation of motion for the entire system can be written in terms of Adjacency and Degree matrices.



And in terms of the Graph Laplacian the System equation is:



This gives a compact form for the behavior of the multi-agent system including robot positions and communication links between robots that guarantees the system will converge. We will now adjust the equation to prevent inter-robot collisions.

***3.2 Edge Tension Energy Minimization***

The previous equation of motion solves the problem of achieving robot rendezvous at a single point. We can introduce weight functions that repel the agents as they get close to each other to prevent them from colliding and maintain a constant inter-agent distance. This approach can be found in [1] and [2].



The weights are functions of the Euclidean norm of the positions between two adjacent agents. The distance is defined as:



We would like to derive our weight functions to prevent agent collisions and maintain constant inter-robot distances. To begin our derivation, we can define a virtual tension energy between two agents as  which is a function of the distances between two agents.

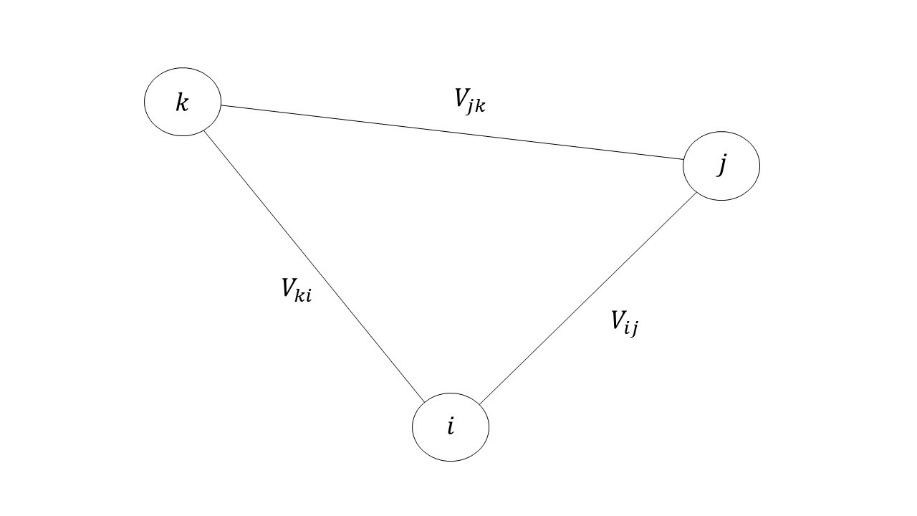


Figure 3.1. Shows an example of a multi-robot system consistent of agents I-j-k with inter agent tension energies. The total energy of the system can be written as:



We divided the sum in half because the System’s Graph is undirected and therefore:



We can rewrite our system equation as a gradient descent algorithm where an agent position is found to minimize :



The desired outcome is achieved because the derivative of the total energy with respect to time is always negative and therefore the energy is always decreasing.



Now the challenge is to find weight functions and objective functions for the energies that would make our gradient descent equation and our consensus equation equivalent. A straightforward way is to choose an energy function which is a function of the inter-robot distances and has distinct global minimum which the gradient descent equation can use to compel robot positions towards. For these reasons, we chose a quadratic function of the form:



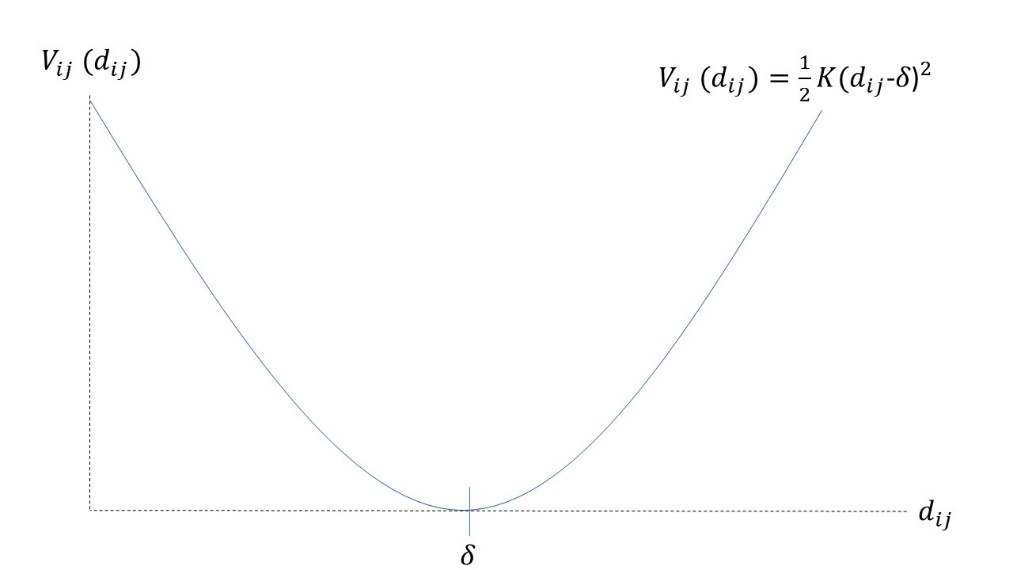


Figure 3.2 shows the form of the energy function. The tension energy function is quadratic and the global minimum of the function can be found at. We can use a gradient descent algorithm to find the function’s minimum. If we take the derivate of the potential with respect to we get:



We know that because the partial derivate of the Euclidean norm can be shown to be:



Therefore, the system equation can be rewritten as:



The weight functions can be written as:



By writing the control algorithm as a gradient descent, two agents moving away from each other will undergo an attractive force from the positive gradient of . They will likewise be repelled by the negative gradient when . The agent will stop moving when . With our new system equation, we can define new Weighted Adjacency and Degree Matrices. The Weighted Degree Matrix is a diagonal matrix with the element  being sum of the weights where s is the degree of  () if the edge exists. By including the elements of the adjacency matrix we ensure that links that do not exist are not included in the sums. We also do not include values where because the link connecting an agent to itself is meaningless in our application This definition for weighted degree matrix is found ins [2].



The Weighted Adjacency Matrix has the same structure as the Adjacency Matrix except the connected elements where are replaced with our weight functions .







Our Weighted System equation can be defined in terms of the Weighted Graph Laplacian matrix which is difference between the weighted Degree Matrix and Weighted Adjacency Matrix:



This again gives a compact description for the behavior of the system:



The above equation describes the behavior of the multi-robot system where each robot has an equation of motion in the form of:



And the total system is guided by the gradient descent algorithm:

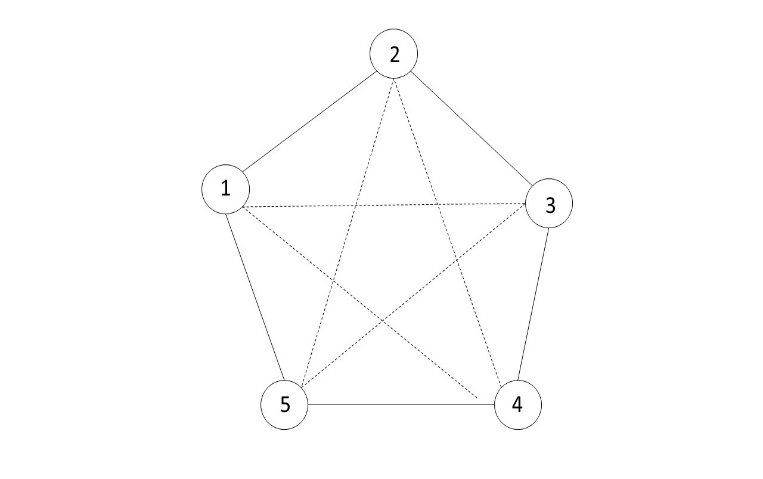


Where the total energy is minimized when the inter-robot distances are equal to the prescribed distance . This allows us to space the robots apart and maintain that distance under perturbations and if the graph is connected. However, we can see that not all robots can be the same distances from all other robots simultaneously which means the total energy of the system will never actually reach zero. To control for a formation, we will need to restrict the allowed edges between vertices. This can be done creating custom Graph Laplacians that define our target formations. In the following section, we will describe how we can choose values for the Adjacency and Degree matrix to create certain shapes with the multi-robot system.

***3.3 Formation Matrices***

We looked at several graph structures and their relationships to robot formation in the context of the control scheme we have developed. Based on the definitions for graphs discussed in section 3.1 we can define Graphs that would give us certain geometric formations. We will proceed to build regular polygons with increasing number of vertices.

N = 5



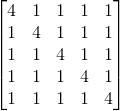
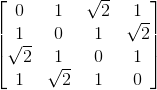
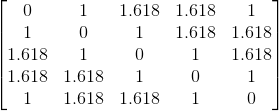


Figure 3. 3 shows the graph for a regular pentagon and its associated Graph Laplacian. The off diagonal elements are those of the Adjacency matrix while the diagonal elements are those of the degree matrix according to equation 6. As an example, the n=5 case describes a pentagon. The first row shows the connections to the first vertex. There will be 4 other vertices connected to it which is represented by the 4 on the degree element and the 4 one’s on the adjacency elements. The same applies to the 4 other vertices. This notation gives us information about whether a connection exists or not but we would like to expand our definition of A to include prescribed distances based on what geometric formation we would like to achieve. We can redefine the A matrix where the non-zero elements contain the desired distance for each robot. For unit distance , the n =2 and 3 cases remain the same but for n = 4 the cross diagonal distances are now .



For a regular pentagon where n = 5 the diagonals will have a length of 1.618. And the adjacency matrix is:



There is a general formula for the length of the diagonals:





We can now define an algorithm for defining our weight functions for higher orders of n.

For elements where, , on the first row or on the last row:



For the elements where , , or where are adjacent to the corner elements:



Now our weights can be written as functions of the elements in of this adjacency matrix.



This determines the weight functions for robots that are adjacent to each other in the regular polygon and for the first cross-diagonal robots. For robot systems composed of less than 6 robots the graph will fully connected. For larger systems, there will be gaps in the connection of graph that will need to account for the system becomes tangled.

***3.4 Avoiding False Formations***

When defining the Weight Matrices for our robot formations, we established connections between adjacent robots with edges corresponding to polygon sides and the cross diagonal edges for regular polygons. This means that the graph is fully connected for up to N = 5 robots. For larger systems, some of the vertices in the graph are disconnected from robots outside of their immediate vicinities. For larger collections of robots, all the inter-robot target distances could be achieved without achieving the desired polygon formation. This gives a false formation and needs to be accounted for.

To compensate for this, we establish temporary connections between robots that are not already connected by the Graph Laplacian. The temporary connections can be activated whenever unconnected robots move within in a certain activation distance of each other. The temporary connection creates an energy tension of the same form as the Graph’s tension parabolic function but drops to zero beyond the vertex of the parabola so that it creates a repulsive force below that activation distance but does not attract beyond that activation distance.

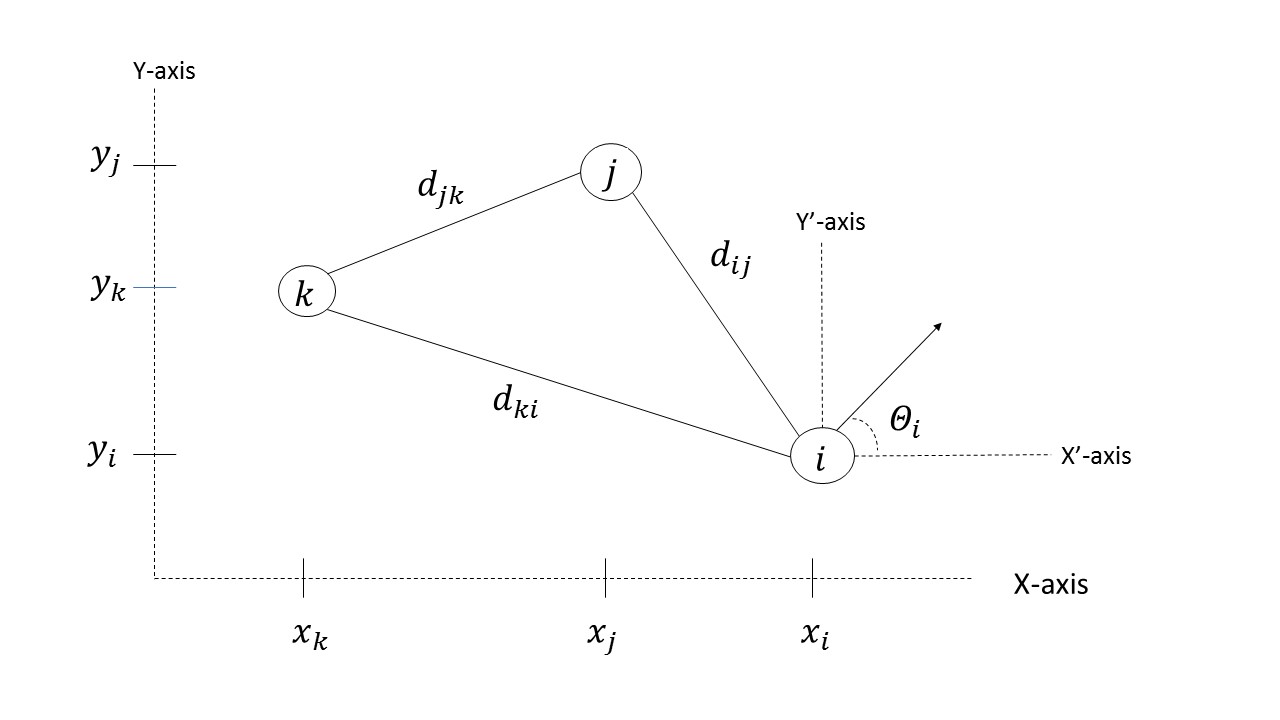
 If and

 If 

We do not know what the activation distance should be beforehand for a given multi-robot system. Each robot requires a separate target distance to create regular polygon formations for larger number of robots. We can dynamically reset that activation distance by increasing it every time it is activated. Evetime robots that should not be in proximity to each get closer to each other than the activation distance, that distance increases. Therefore, every successive false attempt to achieve a formation the system spreads out a small amount until the system is spread out enough to reattempt a formation. The activation distance is capped so that it does not increase indefinitely and cause the system to diverge. The results of this technique are described in Chapter 4.

***3.5 Removing Dependence on Absolute Position***

Let us assume that our multi-agent system is composed of n robots and lets us also consider robots  , and  for now. We will consider robot  to be the primary robot under analysis.  This robot has a direction heading defined by .

**

*Figure 3.4 shows robots*  , and *with their respective coordinate positions**,*  *and* *with respect to fixed frame* *. A rotation-less frame is defined as* *and is fixed to the body of robot* *, which is the robot we will perform analysis on. Robot*  *has a heading vector which is angle* *with respect to the x’-axis.*

We can see from the figure that the difference between horizontal coordinates and  can be written in terms of the Euclidean distance between the robot under consideration and the angle made between the x ‘-axis and the direction to that robot:

 (1)

We can replace the terms  in our weighted consensus equation with equation (1).

 (2)

The angle  is the equal to the robot ’s direction heading and the angle between that angle and the angle the robot needs to turn to face robot .

 (3)

If we substitute equation (3) into equation (2) and invoke the trigonometric identity for angle sums, we get:

 (4)

 (5)

We can rearrange the equation so that we can isolate the terms involving the parameters dependent on a fixed reference frame:

 (6)

We can repeat this process for our vertical terms using the trigonometric identities for the sin(x) function:

 (6)

As with our previous model the heading a robot needs to move towards to satisfy the consensus equation will be induced by the outputs of the consensus equation  and . This heading is angle  and is defined to be with respect to the x axis of the rotation-less reference frame F’. If the robot does not need to move, then these outputs are zero and heading desired heading is not defined.

 (7)

The required angle  is equal to the heading of the analyzed robot plus some angle the robot needs to turn to achieve the required angle.

 (8)

If we substitute equations (5), (6) and (8) into (7) we get:

 (9)

We can once again invoke trigonometric identities to change the LHS:

 (10)

We would like to write our control equation independent of variables referencing the rotation-less frame F’ on robot  or the space fixed frame F. If we compare terms in equation 10 we can see that:

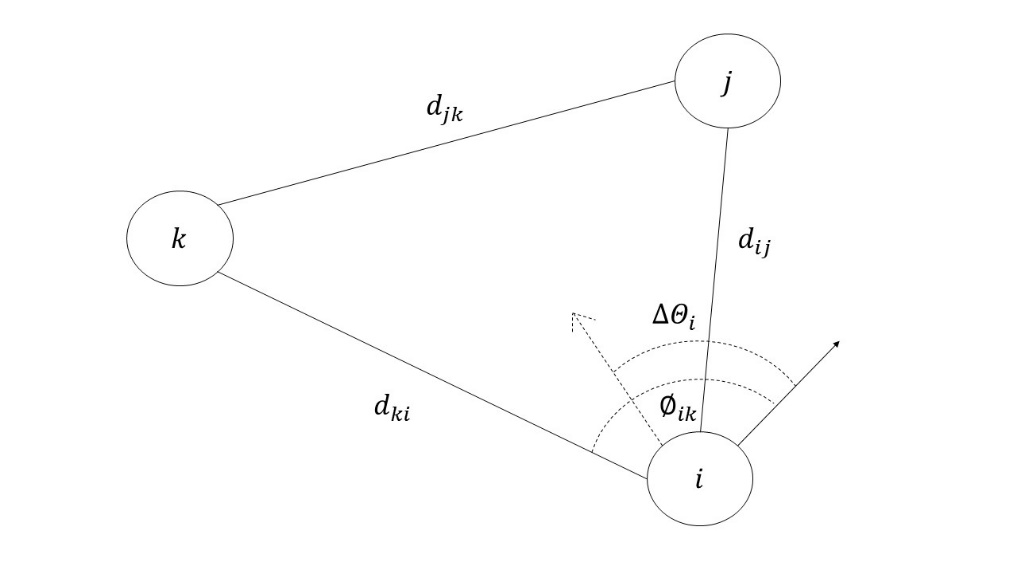
 (11)

 (12)

Therefore, the angle the robot will have to turn to be on the correct heading which will take it to satisfy the gradient descent from section 3.2 is:

 (13)

We can see that equation 13 is depending only on inter robot distances which can be taken from distance sensors, and the angle the neighboring robots make with robot ’s heading. With this information, we can continually update the required heading of each robot in a decentralized and frame independent manner. Figure 3.5 below shows the finally from for the robot network with distance and angel inputs independent of a stationary frame.



This method, however, does not give us the required speeds of each robot. We need a way to tell a robot to stop when it has reached its target location and not move when it is oriented in the wrong direction. One approach we can take is make the speed of the robots proportional to the tension energy between its neighbors. By doing so, as the energy becomes large, the robots will speed up and as the energy approaches zero they will slow down. We will also cap the allowed speed to prevent the system from diverging.





This method was inspired from reading on the adaptive gradient descent algorithm where the iteration rate becomes a function of the gradient instead of a constant [15]. However, instead of using the gradient we are using the edge weight tension energy to change the iteration rate. The information collected from and can be used to update the positions of all robots in our simulation.

To summarize, we can use distance sensors on the robots to gather information on the inter robot distance . If we have several sensors lining the perimeter of the robot we can determine which direction the neighbors are in with respect to a robot’s heading. Using this information, we can calculate , the angle the robot will need to rotate to achieve the desired heading. The desired heading will drive towards satisfying the weighted consensus equation.

**3.6 Obstacle Avoidance**

Now that we have method for achieve a set formation based on the prescribed graph and edge weights for inter-robot distance, we would like to expand our controller to allow the system to avoid obstacles as it moves through a space. To this we can add new rotation commands to angle updates for our robots. The rotation commands are calculated in the same way as in equation (13):



In the above equation,is the distance from robot to an obstacle. Also, is the angle difference between the direction to the obstacle and the robot’s heading. The variables are represented graphically in the figure below:

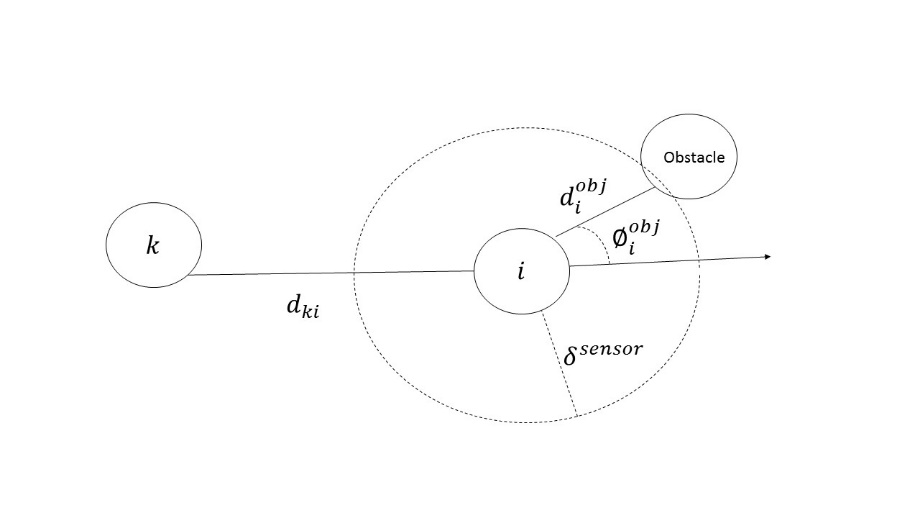


Figure 3.6 above shows a model of an agent interacting with an obstacle and its respective inputs. Finally,is the weight of the edge created between the object and the robot. The term is summed over the number of objects in range of robot .The weights are activated when the robot is in the range of the object:

 If and

 If 

In our weight equation, is the range of the robot’s proximity sensors. We chose this scheme because the value will have a repelling effect on the robot’s equation of motion with respect to the obstacle when the obstacle is within the range of the robot’s sensor but will be zero or non-attractive when it is beyond the range of the sensors.

**Chapter 4: Experimental Results**

4.1 Simulation controller:







4.2 Formation Matrices:

4.3 Test Cases:

Convergence to any point:

Convergence to a target:

Convergence to a target with Obstacles:

Movement through a row of obstacles

Movement through a grid of obstacles:

Movement through moving Obstacles:

4.5 Parameters:

K

N

Rise

Division

Multiplier

4.6 Results

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